***Exercises***

*In this exercise set, use the method of Lagrange multipliers unless otherwise stated.*

**1.** Find the extreme values of the function *f* (*x, y*)= 2*x* + 4*y* subject to the constraint *g*(*x, y*)= *x*2 + *y*2 − 5 = 0.

**(a)** Show that the Lagrange equation ∇*f* = *λ*∇*g* gives *λx* = 1 and *λy* = 2.

**(b)** Show that these equations imply *λ* ≠ 0 and *y* = 2*x*.

**(c)** Use the constraint equation to determine the possible critical points (*x, y*).

**(d)** Evaluate *f* (*x, y*)at the critical points and determine the minimum and maximum values.

**2.** Find the extreme values of *f* (*x, y*)= *x*2 + 2*y*2 subject to the constraint *g*(*x, y*)= 4*x* − 6*y* = 25.

**(a)** Show that the Lagrange equations yield 2*x* = 4*λ*, 4*y* = −6*λ*.

**(b)** Show that if *x* = 0 or *y* = 0, then the Lagrange equations give *x* = *y* = 0. Since *(*0*,* 0*)* does not satisfy the constraint, you may assume that *x* and *y* are nonzero.

**(c)** Use the Lagrange equations to show that *y* = − *x*.

**(d)** Substitute in the constraint equation to show that there is a unique critical point *P*.

**(e)** Does *P* correspond to a minimum or maximum value of *f* ? Refer to Figure 11 to justify your answer. *Hint:* Do the values of *f (x, y)* increase or decrease as *(x, y)* moves away from *P* along the line *g(x, y)* = 0?

*In Exercises 4–13, find the minimum and maximum values of the function subject to the given constraint.*

**4.** *f* (*x, y*)= 2*x* + 3*y*, *x*2 + *y*2 = 4

**5.** *f* (*x, y*)= *x*2 + *y*2, 2*x* + 3*y* = 6

**6.** *f* (*x, y*)= 4*x*2 + 9*y*2, *xy* = 4

**7.** *f* (*x, y*)= *xy*, 4*x*2 + 9*y*2 = 32

**8.** *f* (*x, y*)= *x*2*y* + *x* + *y*, *xy* = 4

**9.** *f* (*x, y*)= *x*2 + *y*2, *x*4 + *y*4 = 1

**10.** *f* (*x, y*)= *x*2*y*4, *x*2 + 2*y*2 = 6

**11.** *f* (*x, y, z*)= 3*x* + 2*y* + 4*z*, *x*2 + 2*y*2 + 6*z*2 = 1

**12.** *f* (*x, y, z*)= *x*2 − *y* − *z*, *x*2 − *y*2 + *z* = 0

**13.** *f* (*x, y, z*)= *xy* + 3*xz* + 2*yz*, 5*x* + 9*y* + *z* = 10

**15.** Find the point (*a, b*)on the graph of *y* = *ex* where the value *ab* is as small as possible.

**16.** Find the rectangular box of maximum volume if the sum of the lengths of the edges is 300 cm.

**17.** Find the point *(x*0*, y*0*)* on the line 4*x* + 9*y* = 12 that is closest to the origin.

**18.** Show that the point *(x*0*, y*0*)* closest to the origin on the line *ax* + *by* = *c* has coordinates

*x*0 = *, y*0 =

**19.** Find the maximum value of *f (x, y)* = *xayb* for *x* ≥ 0*, y* ≥ 0 on the line *x* + *y* = 1, where *a, b >* 0 are constants.

**20.** Show that the maximum value of *f (x, y)* = *x*2*y*3 on the unit circle is .

**21.** Find the maximum value of *f (x, y)* = *xayb* for *x* ≥ 0*, y* ≥ 0 on the unit circle, where *a, b >* 0 are constants.

**22.** Find the maximum value of *f (x, y, z)* = *xaybzc* for *x, y, z* ≥ 0 on the unit sphere, where *a, b, c >* 0 are constants.

**23.** Show that the minimum distance from the origin to a point on the plane *ax* + *by* + *cz* = *d* is

.